Tanta university
Faculty of engineering
Computer and automatic control department

Computer graphics course
Second year students
Sheet 12, Date : --/--/2---

Sheet 12 solution

1. The center of the object is not located at the origin of the frame (world frame=object frame here). In OpenGL, the Affine scaling transformation matrix is defined so that the scaling is done around the origin of the frame. Hence, to do the required effects we need the following transformation in order:

   - Translation so that the object center is located at the frame origin
   - Scaling to shrink the object
   - Translation back so that the origin of the scaled object is located at its previous location

   Knowing that in formatting the Affine transformation matrix for successive transformations, The accumulated transformation matrix is the multiplication of the transformation matrices starting with the last transformation at the left

   $$T = T_{back\ to\ the\ original\ center} \times T_{dcaling} \times T_{moving\ the\ center\ to\ the\ origin}$$

   $$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   $$= \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Given the column data vector for a point P

   $$P_{before\ tranformation} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 1 \end{bmatrix}$$

   The new column data location vector of the transformed point using a transformation matrix M (the model-view matrix in this case) is:

   $$P_{afetr\ tranformation} = M \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 1 \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ 1 \end{bmatrix}$$

   And the inverse transformation is as follows:

   $$P_{before\ tranformation} = M^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 1 \end{bmatrix}$$

   Applying these to the two points we have

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For the point (0,0,10)

$$new\ location = \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \\ 1 \end{bmatrix}$$

For the point (0,0,5)

$$new\ location = \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \end{bmatrix}$$

The points move to [0, 0, 10] and [0, 0, 6]. This is as expected since the center of the object does not move, and the point [0, 0, 5] moves towards the center (shrinks).

3. The order of rotation and shrinking is arbitrary meaning that doing any of them first then the doing the secand adoes not affect the final result. So , one solution is Shrink towards the object center then rotate clockwize arround z

$$T = T_{rotation} \times T_{shrinking}$$

$$T = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where θ=-(i*15) ; i=0,1,3,… the number of the frame

4. The combined transformation matrix or the current transformation matrix(CTM) is as follows
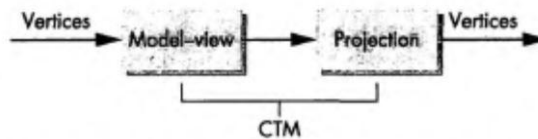
And the combined transformation matrix is
$$T_{Combined} = T_{Projection} \times T_{Model-View}$$
Note the order of the multiplication, this come from the fact:
$$v_{Middle} = T_{Model-View} \times v_{Input}$$
$$v_{output} = T_{Projection} \times v_{Middle} = T_{Projection} \times T_{Model-View} \times v_{Input}$$
Where v is a vector. And the same is applicable to points or vertices
$$T_{Combined(CTM)} = T_{Projection} \times T_{Rotation} \times T_{Scalling}$$

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$$T_{Combined(CTM)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/2 & 0 \end{bmatrix} \times \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$